Bose-Einstein Condensation in Compactified Spaces

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Abstract

We discuss the thermodynamic potential of a charged Bose gas with the chemical potential in arbitrary dimensions. The critical temperature for Bose-Einstein condensation is investigated. In the case of the compactified background metric, it is shown that the critical temperature depends on the size of the extra spaces. The asymmetry of the "Kaluza-Klein charge" is also discussed.

1 Introduction

Recent progress in the unification of gauge interactions has been made in higher dimensions. The original idea of Kaluza and Klein [1] is reviving after about a half century.

In the framework of the so-called Kaluza-Klein theories, gauge fields and symmetries are interpreted as the part of the gravitational field and the symmetries of extra spaces respectively.[2] The revival of Kaluza-Klein theories is activated by the development of supergravity theories.[3] However, many difficulties have been pointed out: such as the chiral fermion problem, unfavorable quantum nature. In many cases, it is well known that the introduction of primary gauge fields can solve the problems.[4] As a result of recent progress in ten-dimensional superstring theories,[5] it is suggested that the gauge fields may be contained in the higher-dimensional theories,

On the other hand, the source of the gauge field, that is, the charge, is a conserved quantity if there is no symmetry-breaking. In our universe, there is no reason why the certain charge asymmetry should vanish from the beginning (of the universe). In the presence of the charge asymmetry, a Fermi gas becomes strongly degenerate at low temperatures. For a Bose gas, under the same circumstances, it is expected that Bose-Einstein condensation takes place. The effect of the charge-asymmetry may also play an important role in the phase transitions in very early universe.[6]

In our previous paper,[7] the finite density effects for Dirac femion fields in higher dimensions are discussed. The compactification in Kaluza-Klein theories

may be also influenced by the behavior of the bosonic matter fields. Especially in supersymmetric theories, the contribution of bosonic fields as well as fermionic matter fields is important.

In the present paper, we consider the properties of a charged Bose gas with non-zero chemical potential and derive the expressions for the critical temperature for Bose-Einstein condensation in arbitrary dimensions and in "Kaluza-Klein background" geometries. The general expression for the thermodynamic potential of a charged scalar boson field is obtained in §2. We also explain how we can determine the critical temperature for condensation in arbitrary flat space. The generalization to (partly) compact spaces is made in §3. The charge stated above is the primary one in the higher dimensions. We can consider the charge induced by the compactification, i.e., by the Kaluza-Klein scheme, and its excess in the universe. In §4, we discuss them, and in particular, investigate whether condensation occurs or not. Finally, §5 is devoted to discussion.

2 Critical temperature in flat arbitrary dimensions

Recently, several authors discussed the expressions of the thermodynamic potential Ω for a Bose gas with mass M and chemical potential μ in flat d-dimensional space at temperature $T = \beta^{-1}$.[8, 9] In this section, we would like to get them by another way which was used for a fermion field in the previous paper.[7]

Allen[8] showed that the thermal partition function can be derived from one-loop effect in a quantum field theory with imaginary time.[11] It was shown that the vacuum energy which needs to be regularized is distinguished from others. Granted that the vacuum energy can be discarded, we find the thermodynamic potential for a charged scalar boson field by using the heat-kernel(-like) method:[7]

$$
\begin{aligned}
\Omega &= -\frac{V_d}{(4\pi)^{1/2}} \int_0^\infty dt\, t^{-3/2} \int \frac{d^d\mathbf{k}}{(2\pi)^d} 2 \sum_{n=1}^\infty \cosh(n\beta\mu) \exp\left\{ -t(\mathbf{k}^2 + M^2) - \frac{\beta^2 n^2}{4t} \right\} \\
&= -\frac{V_d}{(4\pi)^{(d+1)/2}} \int_0^\infty dt\, t^{-(d+1)/2-1} 2 \sum_{n=1}^\infty \cosh(n\beta\mu) \exp\left\{ -tM^2 - \frac{\beta^2 n^2}{4t} \right\} \\
&= -2\frac{V_d}{(4\pi)^{(d+1)/2}} \beta^{-(d+1)} \sum_{n=1}^\infty \cosh(n\beta\mu) 2 \left(\frac{2\beta M}{n} \right)^{(d+1)/2} K_{(d+1)/2}(\beta M n), \quad (1)
\end{aligned}
$$

where V_d denotes the volume of d-dimensional space. (Compare with (2.10) in Ref. [7]).)

The first line of (1) was originated from the zeta function regularization method.[7] This form of the expression will turn out to be useful in the later section.

In order to evaluate the thermodynamic potential as for this expression Ω, we must make the expansion for Ω in terms of infinite series, i.e., high-temperature expansions. Nevertheless, naive procedure of expansion and resummation is

liable to miss the term which has come from the branch point singularity at $\mu = M$ in the complex μ-plane. For instance, the thermodynamic potential for a charged scalar field in even d-dimensional space contains the following form:[8]

$$\Delta\Omega = (-1)^{(d-1)/2} \frac{\pi V_d T}{(4\pi)^{d/2}\Gamma((d+2)/2)} (M^2 - \mu^2)^{d/2} \,. \tag{2}$$

Thus, we should be careful when we calculate the high-temperature expansions of some thermodynamical quantities. In the present paper, we pay attention only to Bose-Einstein condensation and the critical temperature for it, and one can avoid here the complicated issue as one will see later.

In order to examine the property of Bose-Einstein condensations, we show the charge density of the system in the familiar form as follows:

$$
\begin{aligned}
\rho &= -\frac{1}{V_d}\frac{\partial\Omega}{\partial\mu} \\
&= \frac{2}{(4\pi)^{(d+1)/2}}\beta^{-d}\sum_{n=1}^{\infty} n\sinh(n\beta\mu)2\left(\frac{2\beta M}{n}\right)^{(d+1)/2} K_{(d+1)/2}(\beta M n) \\
&= \frac{2M^d}{(4\pi)^{d/2}\Gamma(d/2)}\int_1^{\infty} dx\,(x^2-1)^{(d/2)-1}\left\{\frac{1}{e^{\beta(Mx-\mu)}-1} - (\mu\to-\mu)\right\}, \tag{3}
\end{aligned}
$$

for $d > 0$. At the step into the third line, we used the integral representation

$$K_\nu(z) = \frac{\sqrt{\pi}(z/2)^\nu}{\Gamma(\nu+1/2)}\int_1^{\infty} e^{-zx}(x^2-1)^{\nu-1/2}dx \,. \tag{4}$$

It is easy to see that the final form of (3) is equivalent to

$$\rho = \int_0^{\infty} \frac{d^d\mathbf{k}}{(2\pi)^d}\left[\frac{1}{\exp\{\beta(\sqrt{\mathbf{k}^2+M^2}-\mu)\}-1} - (\mu\to-\mu)\right]. \tag{5}$$

Here, we briefly review the argument for the critical temperature.[9] Above some critical temperature T_c, we can always find $\mu < M$ satisfying (5) if ρ fixed. Below T_c, no such μ can be chosen and we fail in reading (4) as it is. Equation (5) must be interpreted as the charge density in the $\mathbf{k} \neq 0$ state. In other words, if (5) remains finite in the limit $\mu \to M$, the rest of the charge must condensate at the $\mathbf{k} = 0$ state. Therefore, we have only to know the behavior of the expression of the charge density in the limit $\mu \to M$ in order to know the critical value for the condensation; furthermore, we can keep away from the complication of $\Delta\Omega$ in (2) at least in flat d dimensions.

We observe that the charge of the $\mathbf{k} \neq 0$ state expressed by (3) and (5) is finite even in the limit $\mu \to M$ provided $d > 2$. One can find the critical value T_c satisfies the following relation at $\mu = M$ in the high-temperature limit:

$$\rho = \frac{2}{\pi^{(d+1)/2}}\Gamma\left(\frac{d+1}{2}\right) T_c^{d-1} M\,\zeta(d-1), \tag{6}$$

where $\zeta(z)$ is Riemann's zeta function. Hence, we find

$$T_c = \left[\frac{\pi^{(d+1)/2}\rho}{2\Gamma((d+1)/2)\zeta(d-1)M}\right]^{1/(d-1)}, \tag{7}$$

3

and below T_c, $\mathbf{k} = 0$ states cannot carry the charge of scalar particles, so we conclude that there occurs condensation. The physical interpretation of condensation, especially for massless particles, can be found in Ref. [4]. So far we gave an overview of Bose-Einstein condensation in flat space, but around here, we shall turn to discuss condensation in compact spaces.

3 Partly compactified space

In this section, we treat the scalar fields in the background geometry $T \times R^d \times S^N$, which has been discussed frequently as the model space of non-abelian Kaluza-Klein theories. As in the fermion case,[7] we can write down the expressions of thermodynamic potential for the scalar boson as follows:

$$
\begin{aligned}
\Omega &= -\frac{V_d}{(4\pi)^{(d+1)/2}} \int_0^\infty dt\, t^{-(d+1)/2-1} 2 \sum_{n=1}^\infty \cosh(n\beta\mu) \\
&\quad \times \sum_{\ell=0}^\infty d_\ell \exp\left\{ -t(\omega_\ell^2 + M^2) - \frac{\beta^2 n^2}{4t} \right\} \\
&= -\frac{2V_d}{(4\pi)^{(d+1)/2}} T^{d+1} \sum_{n=1}^\infty \cosh(n\beta\mu) \sum_\ell d_\ell 2 \left(\frac{2\beta\sqrt{\omega_\ell^2 + M^2}}{n} \right)^{(d+1)/2} \\
&\quad \times K_{(d+1)/2}(\beta\sqrt{\omega_\ell^2 + M^2}\, n)\,,
\end{aligned}
\tag{8}
$$

where the degeneracy $d_\ell = \{(2\ell + N - 1)\Gamma(\ell + N - 1)\}/\{\ell!\Gamma(N)\}$, and $\omega_\ell^2 = \ell(\ell + N - 1)/a^2$; a is the scale of S^N. The chemical potential μ has been introduced for an elementary charge of the complex scalar field in $(1 + d + N)$ dimensions here. Therefore the total charge density of this system is

$$
\rho = \sum_\ell d_\ell \int_0^\infty \frac{d^d\mathbf{k}}{(2\pi)^d} \left[\frac{1}{\exp\{\beta(\sqrt{\mathbf{k}^2 + M^2 + \omega_\ell^2} - \mu)\} - 1} - (\mu \to -\mu) \right].
\tag{9}
$$

Here, we used the representation of the modified Bessel function (4) again. The above expressions show that the thermodynanic quantities such as ρ are sum of the ones in R^d for the scalar particles with mass $\sqrt{M^2 + \omega_\ell^2}$. In other words, this allows to interpret for the thermodynamical quantities being merely the sum of ones for each "pyrgon"[12] state up to degeneracy, in general.

Let us investigate condensation in the space $R^d \times S^N$. To derive the critical value for T, we must know the value of μ when condensation occurs. In a curved space, setting $\mu = M$ is not so trivial. However, a physical interpretation in this case is easily obtained to conclude that only pyrgon state which has lowest mass is condensate. (We call even zero mass level a "pyrgon state" in this paper.) Thus, we set $\mu = M$ at the critical point.

If $T \ll M$, all pyrgon states can be treated as non-relativistic. Namely, in

(8) we can use the asymptotic form1 of the Bessel function:

$$K_\nu(z) \xrightarrow{z \gg 1} \sqrt{\frac{\pi}{2z}} e^{-z} \,. \tag{10}$$

Using this, we can get

$$\rho \sim \sum_\ell d_\ell \frac{1}{(2\pi)^{d/2}} T_c^{d/2} (\sqrt{\omega_\ell^2 + M^2})^{d/2} \sum_{n=1}^{\infty} \frac{1}{n^{d/2}} \exp\{-n\beta(\sqrt{\omega_\ell^2 + M^2} - M)\} \,. \quad (T \ll M) \tag{11}$$

Further if we assume $Ma \ll 1$, we immediately recognize that the contribution of all the excited pyrgon states except for $\ell = 0$ is suppressed by Boltzmann factor because of their large masses. Therefore we obtain the expression of ρ when $T \ll M \ll 1/a$:

$$\rho \sim \left(\frac{MT_c}{2\pi}\right)^{d/2} \zeta(d/2) \,. \quad (Ta \ll Ma \ll 1) \tag{12}$$

On the other hand, if we suppose $Ma \gg 1$, (11) becomes

$$\rho \sim \sum_\ell d_\ell \left(\frac{T_c}{2\pi}\right)^{d/2} (\sqrt{\omega_\ell^2 + M^2})^{d/2} \sum_{n=1}^{\infty} \frac{1}{n^{d/2}} \exp\left(-\frac{n\beta}{2M}\omega_\ell^2\right) \,.$$
$$(T \ll M \text{ and } Ma \gg 1) \tag{13}$$

For this case, the approximation scheme is divided into two ways according to he magnitude of (MTa^2).

First, suppose $TMa^2 \ll 1$. This condition is equivalent to $1/Ta \gg Ma \gg 1$. Again highly massive modes are suppressed, then only the contribution of $\ell = 0$ survives. We obtain

$$\rho \sim \left(\frac{MT_c}{2\pi}\right)^{d/2} \zeta(d/2) \,. \quad (T \ll M \text{ and } TMa^2 \ll 1) \tag{14}$$

Next, suppose $TMa^2 \gg 1$. For this case, we can use the formula

$$\sum_\ell d_\ell \exp(-x\ell(\ell + N - 1)) \overset{x \ll 1}{\sim} \frac{\Gamma(N/2)}{\Gamma(N)} x^{-N/2} \,. \tag{15}$$

By use of this, we have

$$\rho \sim \left(\frac{MT_c}{2\pi}\right)^{(d+N)/2} \zeta((d+N)/2) V_N \,. \quad (T \ll M \text{ and } TMa^2 \gg 1) \tag{16}$$

where $V_N = (2\pi^{(N+1)/2}/\Gamma((N+1)/2))a^N$. The next leading term will be small by the factor T/M or $1/TMa^2$ compared with the leading (16).

At high temperature $T \gg M$, there are both non-relativistic and relativistic states since the system contains unlimited massive states. A typical example is

the case that only the $\ell = 0$ state can be relativistic, and others are nonrelativistic. Then the condition is given by $T \gg M$ and $T \ll 1/a$. Whereas only $\ell = 0$ term contributes to ρ, for this time we use another asymptotic form of $K_\nu(z)$

$$K_\nu(z) \xrightarrow{z \ll 1} \frac{2^{\nu-1}\Gamma(\nu)}{z^\nu} \, . \tag{17}$$

Then we obtain

$$\rho \sim \frac{2MT_c^{d-1}}{\pi^{(d+1)/2}} \Gamma\left(\frac{d+1}{2}\right) \zeta(d-1) \, . \quad (1/a \ll M \ll T) \tag{18}$$

Finally, we consider an extreme case, i.e., $T \gg M$ and $T \gg 1/a$. In this case, high-temperature expansion is admitted,[13] and it leads to the following expression using (15) and (17):

$$\rho \sim \frac{2MT_c^{d+N-1}}{\pi^{(d+N+1)/2}} \Gamma\left(\frac{d+N+1}{2}\right) \zeta(d+N-1)V_N \, . \quad (T \gg M \text{ and } T \gg 1/a) \tag{19}$$

To summarize: we can obtain several approximate form of (8) as follows:

$$\text{If } T \ll M \text{ and } T \ll 1/a, \text{ then} \quad \rho \sim \left(\frac{MT_c}{2\pi}\right)^{d/2} \zeta(d/2) \, ;$$

$$1 \ll Ma \ll \frac{1}{Ta}, \quad \rho \sim \left(\frac{MT_c}{2\pi}\right)^{(d+N)/2} \zeta((d+N)/2)V_N \, ;$$

$$M \ll T \ll 1/a, \quad \rho \sim \frac{2MT_c^{d-1}}{\pi^{(d+1)/2}} \Gamma\left(\frac{d+1}{2}\right) \zeta(d-1) \, ;$$

$$T \gg M \text{ and } T \gg 1/a, \quad \rho \sim \frac{2MT_c^{d+N-1}}{\pi^{(d+N+1)/2}} \Gamma\left(\frac{d+N+1}{2}\right) \zeta(d+N-1)V_N \, . \tag{20}$$

4 Effect of Kaluza-Klein charge asymmetry

So far we have paid attention to the condensation of a charged boson. Here the "charge" implies the one in the higher-dimensional theory. h this section, we consider the Kaluza-Klein charge asymmetry. For simplicity, we restrict ourselves to studying the abelian Kaluza-Klein model.

First of all, we show the thermodynamic potential Ω with $\mu = 0$ for a neutral scalar field in the geometry $R^d \times S^1$. Note if $\mu = 0$, we can call it "free energy". The thermodynamic potential Ω is

$$\begin{aligned} \Omega &= -\frac{V_d}{(4\pi)^{(d+1)/2}} \int_0^\infty dt \, t^{-(d+1)/2-1} \\ &\quad \times \sum_{n=1}^\infty \sum_{\ell=-\infty}^\infty \exp\left\{-\left(M^2 + \frac{\ell^2}{a^2}\right)t - \frac{\beta^2 n^2}{4t}\right\} \, , \end{aligned} \tag{21}$$

where a is the radius of S^1.

On the other hand, the pyrgons with $(mass)^2 = M^2 + \ell^2/a^2$ have "Kaluza-Klein charge" which is proportional to ℓ/a. By the analogy of the results found in the preceding sections, we introduce the chemical potential as follows:

$$
\begin{aligned}
\Omega &= -\frac{V_d}{(4\pi)^{(d+1)/2}} \int_0^\infty dt\, t^{-(d+1)/2-1} \\
&\quad \times \sum_{n=1}^\infty \sum_{\ell=-\infty}^\infty \cosh(n\beta\mu_\ell) \exp\left\{ -\left(M^2 + \frac{\ell^2}{a^2} \right) t - \frac{\beta^2 n^2}{4t} \right\},
\end{aligned} \tag{22}
$$

where the relation $\mu_\ell = \ell\mu$ reflects the assumption of the chemical equilibrium among pyrgon states.

The particle number can be derived from Ω:

$$
\begin{aligned}
\rho &= -\frac{1}{V_d}\frac{\partial\Omega}{\partial\mu} = \frac{1}{(4\pi)^{(d+1)2}} T^d \sum_{n=1}^\infty \sum_{\ell=1}^\infty 2n\ell \sinh(n\ell\mu\beta) \\
&\quad \times 2 \left(\frac{2\beta\sqrt{M^2 + \ell^2/a^2}}{n} \right)^{(d+1)/2} K_{(d+1)/2}\left(\beta\sqrt{M^2 + \frac{\ell^2}{a^2}}\, n \right).
\end{aligned} \tag{23}
$$

Incidentally, we can express ρ in a familiar form:

$$
\rho = \sum_{\ell=1}^\infty \int \frac{d^d\mathbf{k}}{(2\pi)^d} \left[\frac{1}{\exp\{\beta(\sqrt{M^2 + \ell^2/a^2} - \ell\mu)\} - 1} - (\mu \to -\mu) \right]. \tag{24}
$$

If the value of ρ increases without any limit when we change the value of μ, We can say that there exists no condensation in this system. In the above case, we can see when $\mu > 1/a$ we cannot give physical meaning to ρ or Ω. Thus, we need to investigate the behavior of ρ when μ increases toward $1/a$.

In order to find whether the summation will converge or not, we look into the behavior of the functions at large ℓ or n.

Since the asymptotic form of the modified Bessel function is

$$
K_\nu(z) \overset{z\to\infty}{\sim} \sqrt{\frac{\pi}{2z}} e^{-z}, \tag{25}
$$

thus, the series (22) becomes asymptotically as follows:

$$
\begin{aligned}
&\sum\sum \frac{V_d}{(4\pi)^{(d+1)/2}} T^d 2\sqrt{\pi}\ell \left(2\beta\sqrt{M^2 + \frac{\ell^2}{a^2}} \right)^{d/2} \\
&\quad \times n^{-(d-1)/2} \exp\left\{ \beta n \left(\ell\mu - \sqrt{M^2 + \frac{\ell^2}{a^2}} \right) \right\}.
\end{aligned} \tag{26}
$$

The prefactor of the exponential in the series turns out to be an increasing function in ℓ. Thus the summation on ℓ will be finite provided that the exponential

function in (26) suppresses the large-ℓ terms. However, a closer looking into the exponential reveals that if $\mu = 1/a$ the exponential is not damped at large ℓ. Accordingly, the sum of the series diverges when $\mu \to 1/a$. After all, we conclude that the Bose-Einstein condensation does not occur in this system.

We can arrive at this conclusion by taking the following facts into consideration; first, there are infinite number of states which come from the compactification; second, at the same temperature, the particle system of the larger mass can include the larger number of particles (e.g., see (6)).

Of course, the assumption of the chemical equilibrium among the pyrgon states plays an essential role in this calculation. The physical argument for this will be given in the next section.

5   Discussion

In this paper, we have investigated the Bose-Einstein condensations in higher dimensions and in the Kaluza-Klein space.

We treated here only the free scalar fields and discussed an abelian charge or a particle number. It might be most interesting to check if these results are still valid when interaction is included. It is obvious that, in order to deal with the decay of the particles and non-equilibrium process, we should reformulate our method in deriving the thermodynamic quantities so as to take into account the various interactions and time-dependence of the background geometry. In particular, the argument for the Kaluza-Klein charge in the previous section will not be valid if we take into account the interaction in an expanding universe where the coupling "constant" itself varies when the radius of the extra space changes. Furthermore, the form of the interactions restricts the transition between pyrgon states, and the naive assumption of the chemical equilibrium is not suitable. This must be taken into account when we make the generalization to non-abelian Kaluza-Klein theories.

Further, the curvature and/or the dynamical behavior of the background geometry will induce the interactions which violate the conservation of the Kaluza-Klein charge; this possibility is pointed out by Orito and Yoshimura.[14] As well, the condition of equilibrium is to be re-considered.

To investigate the cosmological aspects of the behavior of matters, it is appropriate to use the so-called "real-time formalism" or TFD (Thermofield Dynamics)[15] arranged more axiomatically, which has been developed recently. Particularly, irreversible and nonequilibrium processes in the universe attract much attention.[16] It is interesting to study the decay of pyrgons which gives rise to the production of entropy in the universe.[12, 17] Also the evolution of the scale of the compact space and its oscillation around the constant value will be affected by the irreversible process such as the particle production.[18]

We are also interested in the study of the behavior of gauge fields in higher-dimensional theory at finite temperature. Superstring theories imply the presence of the gauge field as "primary field".[5] The finite temperature effect on the break-down of primary gauge symmetries as well as compactifications is ex-

pected to be important in the early universe. It is necessary to investigate the symmetry breaking by the non-trivial Wilson line [19] in the evolving multidimensional universe.

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References

[1] Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin, Math. Phys. **K1** (1921) 966.

O. Klein, Z. Phys. **37** (1926) 895.

[2] E. Witten, Nucl. Phys. **B186** (1981) 412, and references therein.

[3] For a review, M. J. Duff, B. E. W. Nilsson and C. N. Pope, Phys. Rep. **130** (1986) 1.

[4] E. Witten, in *Proc. 1983 Shelter Island Conference*, ed. N. Khuri et al. (MIT Press, 1984).

[5] Selected papers can be found in *Superstrings*, ed. J. Schwarz (World Scientific, Singapore, 1985).

[6] A. Love and S. J. Stow, Nucl. Phys. B243 (1984) 537.

[7] K. Shiraishi, Prog. Theor. Phys. **77** (1987) 1253.

[8] A. Actor, Nucl. Phys. **B256** [FS15] (1986) 689.

H. A. Weldon, Nucl. Phys. **B270** [FS16] (1986) 79.

[9] H. E. Haber and H. A. Weldon, Phys. Rev. **D25** (1982) 502; J. Math. Phys. **23** (1982) 1852.

[10] B. Allen, Phys. Rev. **D33** (1986) 3640.

[11] L. Dolan and R. Jackiw, Phys. Rev. **D9** (1974) 3320.

S. Weinberg, Phys. Rev. **D9** (1974) 3357.

[12] E. W. Kolb and P. Slansky, Phys. Lett. **B135** (1984) 378.

[13] Y. Tosa, Phys. Rev. **D33** (1986) 2326; Phys. Lett. **B174** (1986), 156.

J. S. Dowker, Phys. Rev. **D29** (1984) 2773; Class. Quantum Grav. **1** (1984) 359.

M. B. Altaie and J. S. Dowker, Phys. Rev. **D18** (1978) 3557.

[14] S. Orito and M. Yoshimura, Phys. Rev. Lett. **54** (1985), 2457.

[15] H. Umezawa, H. Matsumoto and M. Tachiki, *Thermofield Dynamics and Condensed States* (North Holand, 1982).

I. Ojima, Preprint RIMS-532, January 1986.

[16] M. Morikawa and M. Sasaki, Phys. Lett. **B165** (1985) 59.

M. Morikawa, Phys. Rev. **D33** (1986) 3607.

M. Sakagami and T. Kubota, Phog. Theor. Phys. **76** (1986) 548.

[17] H. Ishihara, A. Tomimatsu and M. Den, Prog. Theor. Phys. **74** (1985) 1045.

J. Saito, Prog. Theor. Phys. **77** (1987) 322.

[18] K. Maeda, Phys. Lett. **B138** (1984) 269.

P. Gribosky, Phys. Rev. **D32** (1985) 3118.

[19] Y. Hosotani, Phys. Lett. **B126** (1983) 309.

E. Witten, Nucl. Phys. **B258** (1985) 75.